Fuzzy Linear Programming Problem

(Discussion and Application)

Noora Ali Aziz*, M.Sc (Asst.Lecturer)

Abstract

We know that the linear programming model (LPM) is one of the most approach used in operation research and decision making, it represent a mathematical model for finding optimum allocation of resources, to satisfy certain objective, like maximize profit or minimize cost. In (LPM), we consider all coefficients of operation function and constraints are real but in real life problem, the wright hand side of (LP), and coefficients of variables of operation function may be a fuzzy number, so this research deals with explaining fuzzy linear programming model with definitions of required and with operation on fuzzy number, then how to solve fuzzy linear programming.

Keywords: Fuzzy linear programming, triangular and trapezoidal fuzzy numbers, Ranking Function.

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* University of Information Technology & Communications

1. Introduction

Fuzzy linear programming (FLP) represent application of fuzzy set theory in decision making problem, which (LP) represent one important tool in building model and taking decision about problems. The LPP consist of objective function which to be maximized or minimized subject to a set of linear equation called constraints, but when the information about coefficients of operation function and of constraints of right hand side of constraints are uncertain, we need to use fuzzy linear programming, where (Zimmermann) introduced fuzzy in multi objective linear programming (Vila, Verdegay, and Delgado [1989]), introduced a general model for fuzzy linear programming[1,2]. Also (Buchley and Feuring [2000]) introduced evolutionary algorithm solution to fuzzy linear programming, (fuzzy sets and systems, 109[2000], 35 – 53). Also H.R.Maleki, ranking functions and their applications to fuzzy linear programming (Far East J. Appli.Math.4(2002), 283 – 301. Also Zimmermann [1978], introduced fuzzy programming and linear programming with several objective functions, (fuzzy sets and systems). Many other researchers studied fuzzy fractional programming but the concept of fuzzy linear programming on the general level was first proposed and introduced by Tanaketal. The objective of this paper is to explain how to transform fully fuzzy linear programming with triangular numbers, and trapezoidal number, into regular linear programming method through transformation and ranking all definitions. The aim of the research is explaining fuzzy linear programming, and how to solve fuzzy linear programming.

This paper divided into (5) section, section (1) contains the introduction, section (2), we present some preliminaries through five famous definitions of fuzzy and sets, section (3) we introduced fuzzy linear programming, the application of this paper is in section (4) contains two examples, then finally contains the conclusion.

2. Preliminaries

Definition (1): Let (x) be a set of objects, its elements denoted by (x) which is real or complex, but when (x) have fuzzy nature, then it characterized by a function $[\mu_A(x)]$ from $[x \ to \ (0,1)]$, such that[3];

$$\mu_A(x) = 1 \qquad if\ x \in A$$
$$= 0 \qquad o/w$$

Then;

$$A = \{(x, \mu_A(x) | x \in X\}$$

Definition (2): The (α) level $(\alpha - cut)$ set of a fuzzy set (A) is a crisp subset of (X), and it is denoted by[3,4];

$$A_\alpha = \{x \in X |\ \mu_A(x) > \alpha\}$$

Definition (3): A fuzzy set (A) in (X) called convex if[6];

$\mu_A\{\lambda x(1-\lambda)y\} \geq Min\ \{\mu_A(x), \mu_A(y)\} x, y \in X\ and\ \lambda \in [0,1]$ ……..(1)

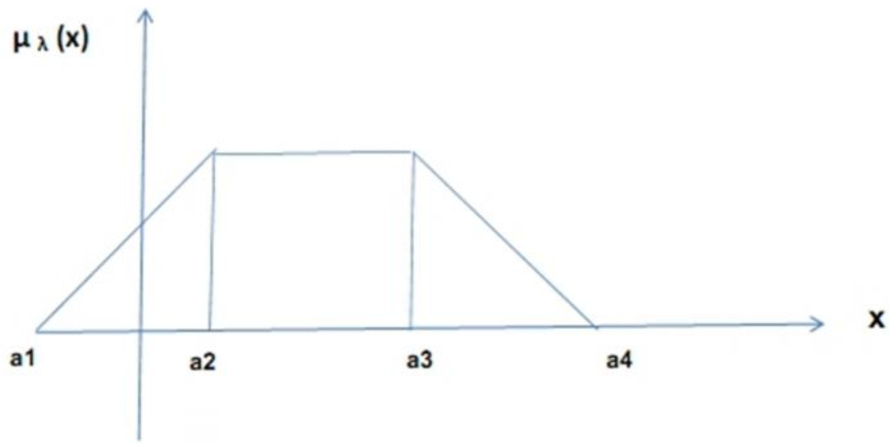
Definition (4): A fuzzy number $(\tilde{A})$ is a convex normalized fuzzy set on the real line R such that[5];

(i). It exists at least one $(x_0 \in R)$ with $[\mu_A(x_0) = 1]$.

(ii). $\mu_A(x)$is piece wise continuous.

Definition (5): The fuzzy number in $(\tilde{A})$ may be triangular and trapezoidal fuzzy numbers, these are defines as in case of trapezoidal[7];

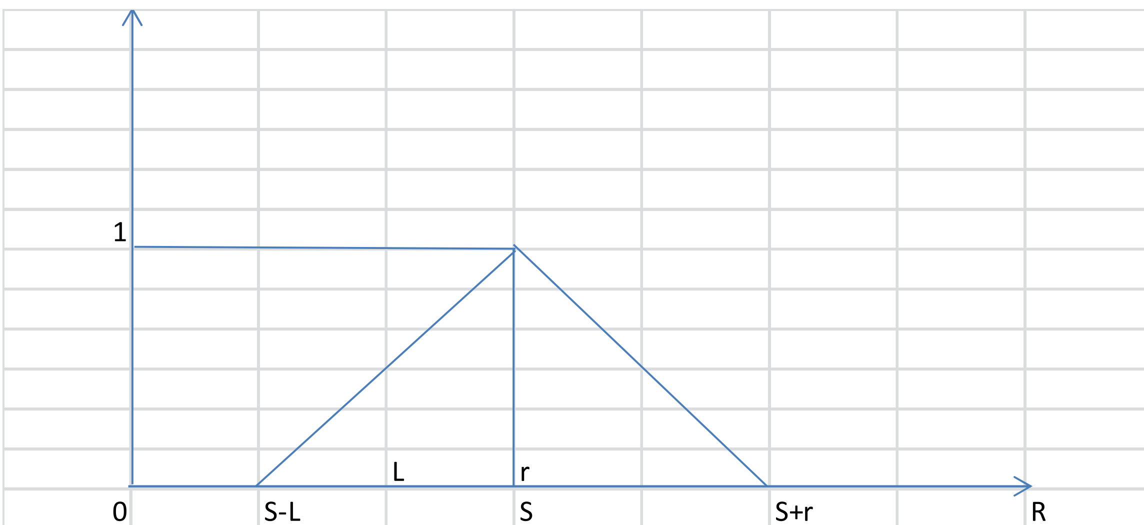
$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x < a_1 \\ \frac{x-a_1}{a_2-a_1} & a_1 \leq x \leq a_2 \\ & a_2 \leq x \leq a_3 \\ \frac{a_4-x}{a_4-a_3} & a_3 \leq x \leq a_4 \\ 0 & x > a_4 \end{cases} \qquad \ldots\ldots\ldots\ldots (2)$$



For this shape the $(\alpha - cut)$ interval is written for all as;

$\alpha \in [0.1], \tilde{A}_0 = [(a_2 - a_1)\alpha + a_1 (a_4 - a_3)\alpha + a_4]$

Where [F(R)] be the set of all trapezoidal fuzzy numbers, while the triangular fuzzy number $(\tilde{A})$ can be represented by three real numbers, (S,L,r) by triangular;



Where [$\tilde{A} = (S, L, r)$], and [$F_1(R)$] is the set of all triangular fuzzy numbers.

Now we assume some mathematical operations on fuzzy numbers;

a. In case of trapezoidal fuzzy number;

let $\tilde{A} = (a_1, a_2, a_3, a_4)$, $\tilde{B} = (b_1, b_2, b_3, b_4)$, be two sets of trapezoidal fuzzy numbers, the operations are[8];

(i): Addition

$\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4$……….(3)

(ii): Subtraction

$\tilde{A} - \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$ …… (4)

(iii): Scalar multiplication

Let $x > 0: x\tilde{A} = (xa_1, xa_2, xa_3, xa_4)$, if $x < 0: x\tilde{A} = (xa_4, xa_3, xa_2, xa_1)$

b. Mathematical operations on triangular fuzzy number;

let $\tilde{A} = (S_1, L_1, r_1)$, $\tilde{B} = (S_2, L_2, r_2)$, be two sets of triangular fuzzy numbers, the operations are[9];

(i): Addition

$\tilde{A} + \tilde{B} = (S_1 + S_2, L_1 + L_2, r_1 + r_2)$ ……………(5)

(ii): Subtraction

$\tilde{A} - \tilde{B} = (S_1 - S_2, L_1 - L_2, r_1 - r_2)$ ……………(6)

(iii): Scalar multiplication

Let $x > 0: x\tilde{A} = (xS_1, xL_1, xr_1)$, if $x < 0: x\tilde{A} = (xr_1, xL_1, xS_1)$ .... (7)

c. Ranking Functions

A convenient method for comparing to fuzzy number is, to use the ranking function which represents a map from [F(R)] into the real line, now we define orders on [F(R)] as follows[10];

$$\left.\begin{array}{l}\tilde{a} \geq \tilde{b}\ iff\ R(\tilde{a}) \geq R(\tilde{b}) \\ \tilde{a} > \tilde{b}\ iff\ R(\tilde{a}) > R(\tilde{b}) \\ \tilde{a} = \tilde{b}\ iff\ R(\tilde{a}) = R(\tilde{b})\end{array}\right\} \quad \ldots\ldots\ldots\ldots (8)$$

Where ($\tilde{a}\&\tilde{b}$) are in [F(R)].

We know that there are many ranking functions for comparing fuzzy number, but we use only linear ranking functions, i.e;

$\tilde{a} \geq \tilde{b}\ iff\ \tilde{a} - \tilde{b} \geq 0$, $\tilde{a} \geq \tilde{b}\ iff\ and\ \tilde{c} \geq \tilde{d}$, then $\tilde{a} + \tilde{c} \geq \tilde{b} + \tilde{d}$, $(\tilde{a}, \tilde{b}, \tilde{c}, \tilde{d} \in R)$

3. Fuzzy Linear Programming

In this section we explain fuzzy linear programming (FLP), in which the coefficients of objective function and the coefficients of constraints and values of right hand side of constraints is fuzzy number of triangular type[11];

$$Max\ \tilde{Z} = \sum_{j=1}^{n} \tilde{c}_j x_j \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (9)$$

s.to:

$$\sum_{j=1}^{n} (S_{ij}, L_{ij}, r_{ij}) x_{ij} \leq (t_i, u_i, v_i) \qquad for\ (i \in N_m) x_j \geq 0 \qquad (j \in N_n)$$

$$A_{ij} = (S_{ij}, L_{ij}, r_{ij}), \qquad B_i = (t_i, u_i, v_i)$$

We can rewrite fuzzy linear programming in equation (9) as;

$$Max\ \tilde{Z} = \sum_{j=1}^{n} \tilde{c}_j x_j$$

s.to:

$$\sum_{j=1}^{n} S_{ij}x_j \leq t_i$$

$$\sum_{j=1}^{n}(S_{ij} - L_{ij})x_j \leq t_i - u_i \qquad \text{……….....(10)}$$

$$\sum_{j=1}^{n}(S_{ij} + r_{ij})x_j \leq t_i + v_i$$

$$for\ (i \in N_m) x_j \geq 0 \qquad (j \in N_n)$$

Here the fuzzy linear programming problem in (10) becomes;

$$Max\ \tilde{Z} = \tilde{C}x \qquad \text{…………………….. (١١)}$$

s.to:

$$Ax \leq b$$

$$x \geq 0$$

Another type of fuzzy linear programming problem in which the variables are trapezoidal fuzzy variables, the coefficients of objective function and right hand side of the constraints are trapezoidal fuzzy numbers[12];

$$Max\ \tilde{Z} = \tilde{C}\tilde{x} \qquad \text{……………………..(12)}$$

s.to:

$A\tilde{x} \leq \tilde{b}$

$\tilde{x} \geq 0$

Where;

$\tilde{b} \in [F(R)]^m$

$\tilde{x} \in [F(R)]^n$

$A \in R^{m\times n}$

$\tilde{C} \in [F(R)]^n$

The third type of fuzzy linear programming problem, is one in which the coefficients of operation function, the coefficients of the constraints and right hand side of constraints are of type fuzzy triangular numbers[13];

$Max\ \tilde{Z} = \sum_{j=1}^{n} \tilde{c}_j x_j$ ….……………………..(13)

s.to:

$$\sum_{j=1}^{n} \tilde{A}_{ij} x_j \leq \tilde{B}_i, \qquad x_j \geq 0$$

$Max\ \tilde{Z} = \sum_{j=1}^{n} \tilde{c}_j x_j$ ………………………… (14)

s.to:

$$\sum_{j=1}^{n} (S_{ij}, L_{ij}, r_{ij}) x_{ij} \leq (t_i, u_i, v_i) \qquad for\ (i \in N_m) x_j \geq 0 \qquad (j \in N_n)$$

And, $A_{ij} = (S_{ij}, L_{ij}, r_{ij})$, $B_i = (t_i, u_i, v_i)$ are fuzzy triangular numbers $[\tilde{c}_j \in F_1(k)]$.

The fuzzy linear programming problem in (14), can be written using operations on fuzzy numbers[14]:

$$Max\ \tilde{Z} = \sum_{j=1}^{n} \tilde{c}_j x_j$$

s.to:

$$\sum_{j=1}^{n} S_{ij} x_j \leq t_i$$

$\sum_{j=1}^{n} (S_{ij} - L_{ij}) x_j \leq t_i - u_i$ ………….(15)

$$\sum_{j=1}^{n} (S_{ij} + r_{ij}) x_j \leq t_i + v_i$$

$for\ (i \in N_m) x_j \geq 0 \qquad (j \in N_n)$

Then problem in (15) can be summarized by;

$Max\ \tilde{Z} = \tilde{C}x$ ………………….. (16)

s.to:

$Ax \leq b$

$x \geq 0$

Where;

$\tilde{C}^T \in [F_1(R)]^n$

$b \in R^m$

$x \in R^n$

R is linear ranking function, problem (16) can be solved using simplex method.

4. Application

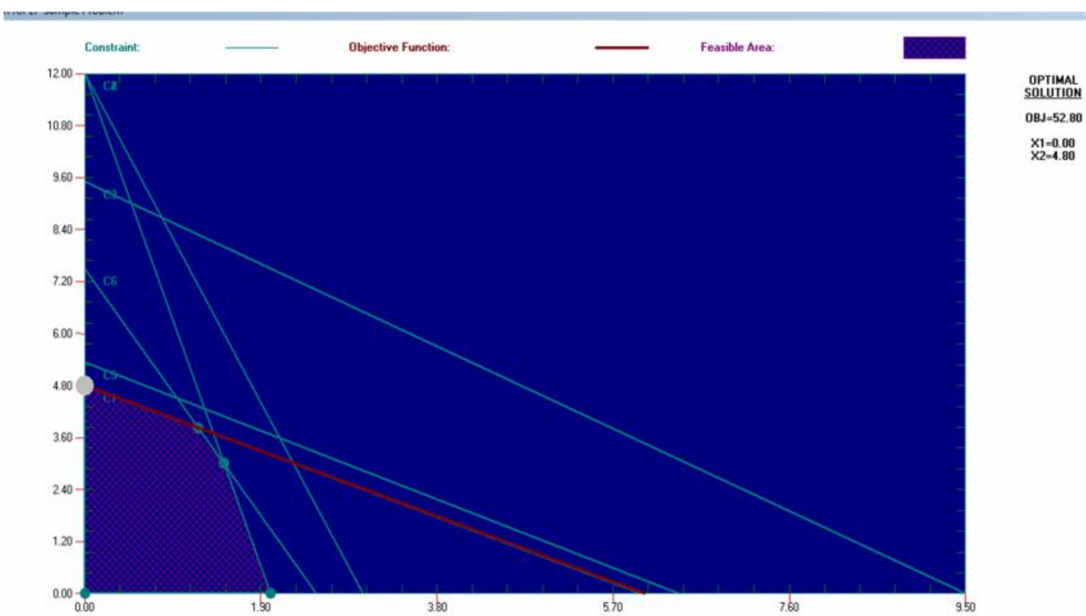
The application here is considered through examples and solving each transformed fuzzy linear programming into ordinary linear programming.

Example (1):

$Max\ \tilde{Z} = (8,5,2)x_1 + (10,6,2)x_2$
s.to:
$(4,2,1)x_1 + (5,3,1)x_2 \leq (24,5,8)$
$(4,1,2)x_1 + (1,0.5,1)x_2 \leq (12,6,3)$
$x_1, x_2 \geq 0$
According to (15) we can write this equation as;
$Max\ \tilde{Z} = 8.75x_1 + 11x_2$
s. to:
$4x_1 + 5x_2 \leq 24$
$4x_1 + x_2 \leq 12$
$2x_1 + 2x_2 \leq 19$
$3x_1 + 0.5x_2 \leq 6$
$5x_1 + 6x_2 \leq 32$
$6x_1 + 2x_2 \leq 15$
$x_1, x_2 \geq 0$

Solving the problem using WinQSB program, we get, ($x_1 = 0, x_2 = 4.8, Z = 52.8$);
and graphically by WinQSB program

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	0	8.7500	0	-0.0500	at bound	-M	8.8000
2	X2	4.8000	11.0000	52.8000	0	basic	10.9375	M
	Objective	Function	(Max.) =	52.8000				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	24.0000	<=	24.0000	0	2.2000	0	26.6667
2	C2	4.8000	<=	12.0000	7.2000	0	4.8000	M
3	C3	9.6000	<=	19.0000	9.4000	0	9.6000	M
4	C4	2.4000	<=	6.0000	3.6000	0	2.4000	M
5	C5	28.8000	<=	32.0000	3.2000	0	28.8000	M
6	C6	9.6000	<=	15.0000	5.4000	0	9.6000	M



Example (2): In the second example, the coefficients of variables in operation functions are in trapezoidal type;

$$Max\ \tilde{Z} = (3,5,8,13)x_1 + (4,6,10,16)x_2$$

s.to:

$$(4,2,1)x_1 + (5,3,1)x_2 \leq (24,5,8)$$

$$(4,1,2)x_1 + (1,0.5,1)x_2 \leq (12,6,3)$$

$$x_1, x_2 \geq 0$$

The model of this example can be written in crisp form using operating in equation (15) for constraints and apply ranking $[R(\tilde{a})]$ for trapezoidal coefficient to transform;

$Max\ \tilde{Z} = (3,5,8,13)x_1 + (4,6,10,16)x_2$

into

$Max\ Z = -18x_1 - 18x_2$

s. to:

$4x_1 + 5x_2 \leq 24$

$4x_1 + x_2 \leq 12$

$2x_1 + 2x_2 \leq 19$

$3x_1 + 0.5x_2 \leq 6$

$5x_1 + 6x_2 \leq 32$

$6x_1 + 2x_2 \leq 15$

$x_1, x_2 \geq 0$

Where;

$R(\tilde{a}) = a^L + a^u + \frac{1}{2}(\beta - \alpha)$

Since each trapezoidal coefficient is, (a, a^L, a^u, β).Then each constraint is transformed by adding slacks variables as;

$Min\ \tilde{Z} = -18x_1 - 18x_2$
s. to:

$4x_1 + 5x_2 + x_3 = 24$

$4x_1 + x_2 + x_4 \leq 12$

$2x_1 + 2x_2 + x_5 \leq 19$

$3x_1 + 0.5x_2 + x_6 \leq 6$

$5x_1 + 6x_2 + x_7 \leq 32$

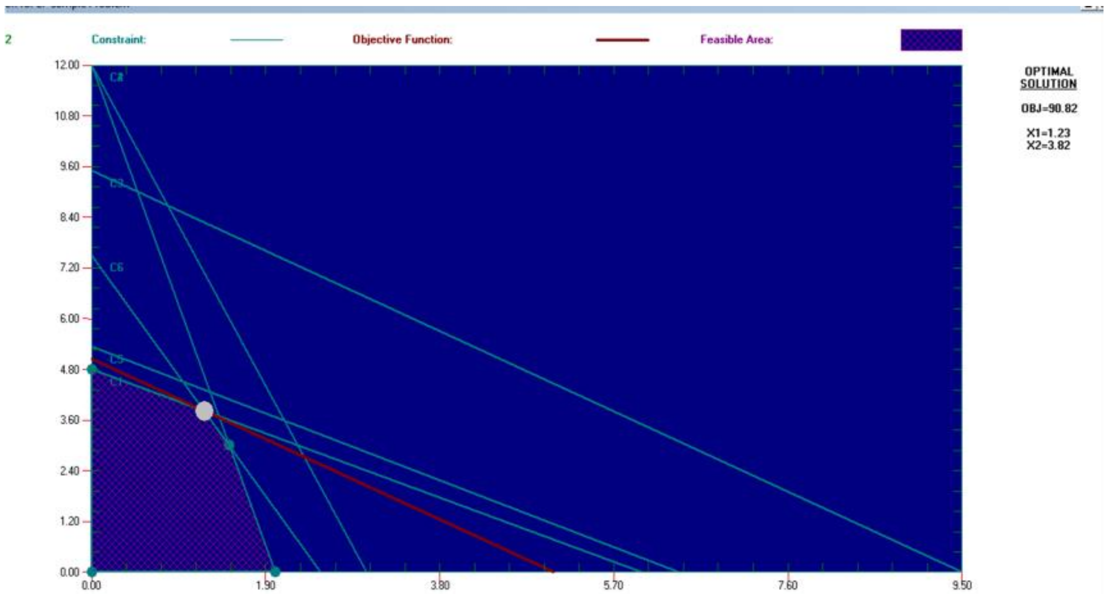
$6x_1 + 2x_2 + x_8 \leq 15$

$x_1, x_2, x_3, x_4, \ldots, x_8 \geq 0$

Then the crisp model solved by simplex iteration, to obtain the optimal solution, Solving the problem using WinQSB program, we get, ($x_1 = 1.23, x_2 = 3.82, Z = 90.82$);

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.2273	18.0000	22.0909	0	basic	14.4000	54.0000
2	X2	3.8182	18.0000	68.7273	0	basic	6.0000	22.5000
	Objective	Function	(Max.) =	90.8182				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	24.0000	<=	24.0000	0	3.2727	21.0000	26.5000
2	C2	8.7273	<=	12.0000	3.2727	0	8.7273	M
3	C3	10.0909	<=	19.0000	8.9091	0	10.0909	M
4	C4	5.5909	<=	6.0000	0.4091	0	5.5909	M
5	C5	29.0455	<=	32.0000	2.9545	0	29.0455	M
6	C6	15.0000	<=	15.0000	0	0.8182	9.6000	15.6923

and graphically by WinQSB program as;



Conclusion

In this paper ranking and certain transformation approach was applied to convert fuzzy linear programming with triangular fuzzy numbers in operation functions and coefficients of constraints , right hand side of constraints using rules explained in equation (15), also using ranking formula for the coefficient of operation function, so this transformations are necessary to transform fuzzy linear programming model into crisp model, to make so using simplex method trivial for solution.

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مشكلة البرمجة الخطية الضبابية: مناقشة وتطبيق

م.م.نورا علي عزيز*

المستخلص

تعتبر البرمجة الخطية احدى اهم المقاربات في بحوث العمليات وصناعة القرار, وتمثل نماذج رياضية لأيجاد التخصيص الأمثل للموارد لأشباع مختلف الحاجات, مثل تعظيم الأرباح أو تقليل الكلف. في نماذج البرمجة الخطية تعتمد كافة المعادلات لدوال العمليات والقيود كقيم حقيقية, الجانب الأيمن من معاملات ومتغيرات البرمجة الخطية لدالة الهدف يمكن أن تكون قيم ضبابية. سيتم في هذا البحث توضيح لنماذج البرمجة الخطية الضبابية مع تعاريفها المطلوبة وكيفية أيجاد حلول لهذه النماذج.

*جامعة تكنولوجيا المعلومات والأتصالات