Behaviour of the parameter of Non-Guassian Seasonal Autogressive Model from first order SAR (1)

(Simulation Study)

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ABSTRACT

This research aims at following the behaviour of Parameter of Non – Gaussion Seasonal Autoregressive Model from first order by simulation. This is done by supposing that model random errors follow Non – Gaussion continuous distribution and by using two comparison tools mean square error MSE (Φ) and mean absolute percentage MAPE (Φ) for the difference cases (seasonal period , sample size , parameter initial values and the sign of parameter initial values).

The main conclusions are (i) MAPE (Φ) and MSE (Φ) values increase when seasonal period is increase , sample size with negative initial value of the parameter and the parameter initial values are approaching zero, and (ii) MAPE and MSE values of parameter Φ decrease when season period is decreased (Regardless of the parameter is positive or negative), parameter initial value is $\pm$ 1 and sample size is increase with positive initial value of the parameter Φ.

1- Introduction:

In 1926, Yule was the first to present autoregressive.

In 1931, Walker studied autoregressive model till P order Then, detailed studies were done autoregressive including both – theoretical and practical forms and method. But research in seasonal time series is still limited, in 1973, Harrison was the first to study seasonal time by studying number and parameters, and season length, and comparing them with Box- Jenkins. Also in 1973, Chatifled and Prothero studied seasonal forecasting and analysed seasonal series with seasonal behvaviour.

The research aims at studying and following the behaviour of a parameter of Non – Gaussion seasonal autoregressive model from first order by using simulator. This is done by supposing that random error of the studied model follow in each simulation experiment one of the following continuous distributions: Uniform, Exponential, Gamma, Beta, Cauchy, Lognormal, Logistic, Laplace. Weibull and pareto; and using the two comparison tools: parameter mean square error MSE (Φ) and using the two comparison tools: parameter mean square error MSE (Φ) and parameter mean absolute percentage error MAPE (Φ) .

For the difference cases (season period, sample size, parameter initial values, and the sign of parameter initial values).

2- Seasonal Autoregressive from first order SAR (1):

2-1: Model Formula:

The general formula of seasonal autoregressive from first order SAR (1) can be written as follows:

$$X_t = \Phi_s X_{t-s} + a_t \qquad \ldots (2.1)$$

Where:

X_{t-is} i=0, 1 bservation values of seasonal time series

S Seasonal length

Φs seasonal autoregressive parameter

2-2: Theoretical Aspects of the model:

a - Stationary:

To make the model stationary, the equation roots should be $\Phi(B) = 0$

That's,

$1 - \Phi_s B^s = 0$ outside the unit circle. This leads to:

$$-1 \leq \Phi_s \leq 1 \qquad \textbf{... (2.2.1)}$$

b- Auto covariance:

The general formula of model auto covariance can be written as follows:

$$g_k = \begin{cases} s_a^2/\left(1-\Phi_s^2\right) & ,K=0 \\ \Phi_s s_a^2/\left(1-\Phi_s^2\right) & ,K=S \\ 0 & ,K=1,2,....,S-1 \end{cases} \quad \textbf{....(2.2.2)}$$

C- Auto correlation:

The general formula of model autocorrelation can be written as follows:

$$\ell_K = \begin{cases} 1 & ,K=0 \\ \Phi_S & ,K=S \\ 0 & ,K=1,2,....,S-1 \end{cases} \quad \textbf{....(2.2.3)}$$

2-3. Estimating Model Parameter:

The model parameter is estimated by Exact Likelihood method, with following formula:

$$\hat{\Phi}_s = \frac{n-s-1}{n-s} \frac{\sum_{t=s+2}^{n} X_t X_{t-s}}{\sum_{t=s+1}^{n} X_{t-s}^2} \quad \text{....(2.3)}$$

3-Simulation:

10 simulation experiments are made. In each experiment, random error a generated as random variables following a certain distribution (see Table (1)). This experiment is repeated 5000 for sample size (50,100,150), for parameter initial values (±0.8, ±0.5, ±0.2) and for seasonal periods (4,12).

Table (1)

General formula for generating variables follows continuous distributions

Distribution	Formula
Uniform U(∝,β)	$a_t = \propto + (\propto - \beta)u$
Exponential Ex p (θ)	$a_t = -\theta.Ln\,(1-u)$
Gamma Gam (∝,β)	$a_t = \beta.Ln\,(\pi^{n}_{i=1} ui)$
Beta Be (∝,β)	$a = y_1 / (y_1 + y_2)$, $y_1 = u_1^{\frac{1}{\pi}}$, $y_2 = u_2^{\frac{1}{\beta}}$, $y_1 + y_2 < 1$
Cauchy Cau (∝,β)	$a_t = \propto + \beta.\tan\,[\pi(u-0.5)]$
Log-Normal Logn (μ,6²)	$a_t = Exp\,[(-2Ln(u_1))^{½} Cos\,(2\pi u_2)]$
Logistic Logi (∝,β)	$a_t = \propto + \beta.Ln(\frac{1}{u} - 1)$
Laplace Lap(∝,β)	$a = \propto -. \beta.Ln[2(1-u)]$
Weibull Wei (∝,β)	$a_t = \propto - \beta\,[Ln(1-u)]^{\frac{1}{\mu}}$
Pareto Par(∝,β)	$a_t = \left[\frac{\mu}{1-u}\right]^{\beta} Exp\,(\frac{1}{\beta})$

The result of simulation are presented in the tables (2-11)

4- Comparison Tools:

The research using the two comparison tools:

a - $MSE(\Phi_s) = \Sigma\ (\Phi - \Phi\hat{i})^2 / R$ …(4.1)

$$b - MAPE(\Phi_s) = \frac{1}{R}\sum_{i-1}^{R}\left|\frac{\Phi - \Phi_i^{\wedge}}{\Phi}\right| * 100 \qquad ……(4.2)$$

Where:

R is Replicate Number

Table (2)

Uniform Distribution **$a_t \sim u(1-2)$**

N		N= 50		N= 100		N=150	
S	**Φ**	**MSE**	**MAPE**	**MSE**	**MAPE**	**MSE**	**MAPE**
4	**-.8**	1.53046	153.7376	1.66625	160.8499	1.719216	163.5559
	-.5	1.577464	251.0286	1.696687	260.4429	1.738271	263.6433
	-.2	1.158633	538.0923	1.225823	553.5483	1.248106	558.5726
	.2	.5720357	378.126	.5913709	384.4931	.5975103	386.4896
	.5	.2449484	98.97482	.2436588	98.72165	.2435924	98.70921
	.8	.05643915	29.6903	.04491715	26.49113	.04249022	25.76607
12	**-.8**	1.145761	133.2605	1.407436	147.8468	1.528452	154.2022
	-.5	1.284027	226.3387	1.550158	284.9089	1.638308	255.9341
	-.2	1.05964	514.4384	1.183004	543.7638	1.220436	552.3337
	.2	.6120088	393.8741	.6128489	391.399	.6112614	390.9058
	.5	.3614755	120.1895	.2835185	106.4854	.2661807	103.138
	.8	.1925571	54.81507	.08882038	37.24831	.06498638	31.86378

Table (3)
Exponential distribution a_t ~ Exp(2)

N		N= 50		N= 100		N=150	
S	**Φ**	**MSE**	**MAPE**	**MSE**	**MAPE**	**MSE**	**MAPE**
4	**-.8**	.1008502	34.62887	0.6361642	28.52714	.05384916	26.84512
	-.5	.1942608	82.8933	.1674922	78.8819	.1603867	78.0194
	-.2	.2463553	240.7645	.2383504	240.1232	.2370896	240.0418
	.2	.2174721	230.121	.2234844	234.9887	.2261201	236.8 694
	.5	.1329158	72.35992	.1361669	73.61688	.1376127	74.08466
	.8	0.04099346	25.1151	.03481232	23.28804	.03359199	22.89611
12	**-.8**	.1689281	46.76237	0.08524452	33.6663	.03641928	29.45 665
	-.5	.2337902	90.97487	.1827786	82.40308	.1670937	79.70251
	-.2	.2656746	247.8949	.2454042	243.1865	.2398287	242.03
	.2	.2334541	235.4719	.2276442	236.6951	.228387	237.8922
	.5	.1700171	80.48749	.1480952	76.59925	.144559	75.78184
	.8	.1046598	39.30177	.05936206	30.29471	.0471831	27.0957

Table (4)
Gamma Distribution a ~ Gam (1,2)

N		N= 50		N= 100		N=150 t	
S	Φ	MSE	MAPE	MSE	MAPE	MSE	MAPE
4	-.8	.4220977	77.54455	.3586347	72.54374	.3364355	70.09392
	-.5	.6891066	163.9744	0. 6970858	165.864	.7004988	166.6479
	-.2	.6794756	410.342	.710057	420.4443	.7209805	423.9847
	.2	.4277658	326.5362	.4459062	333.699	.4519559	336.0301
	.5	.2066759	90.84193	.2085167	91.30633	.2093872	91.50672
	.8	.05183579	28.41964	.0421281	25.6497	.04008367	25.02358
12	-.8	.5177729	87.71037	.4105807	78.27332	.3672047	74.35253
	-.5	.6693449	161.6824	.691609	165.2435	.695603	166.1329
	-.2	.652936	401.7458	.6999897	417.3715	.7143104	421.9929
	.2	.4549207	336.1744	.4578971	338.0803	.4597181	338.8748
	.5	.2870525	106.8004	.2372548	97.36266	.2259338	95.04262
	.8	.1600174	49.78031	.07987463	35.29462	.058985	30.58098

TABLE (5)

Beta Distribution $a_t \sim Be(1,2)$

N		N= 50		N= 100		N=150	
S	Φ	MSE	MAPE	MSE	MAPE	MSE	MAPE
4	-.8	.208134	52.8197	.156949	46.9676	.1402522	44.99056
	-.5	.3903171	121.7768	.376639	120.9884	.3710604	120.6419
	-.2	.4448439	330.0684	.4556191	335.672	.4622365	338.8344
	.2	.3292409	285.8551	.345127	293.3294	.3502463	295.6653
	.5	.1755609	83.62487	.1793722	84.65694	.1808588	85.02986
	.8	.04762751	27.20144	.0348814	24.82673	.03779592	24.29654
12	-.8	.3001679	65.38277	.1932911	52.7554	.1625735	48.7344
	-.5	.4127085	125.2713	.3874359	122.3539	.3774998	121.6999
	-.2	.4412574	327.7397	.4596291	336.692	.4614376	338.4565
	.2	.3456635	291.7461	.3515794	295.899	.3548177	295.5156
	.5	.2324019	95.68861	.2001961	89.37312	.192908	87.81324
	.8	.1346667	45.41812	.07199182	33.47731	.05517217	29.33878

TABLE (6)
Cauchy Distribution $a_t \sim Cau\ (1,2)$

N		N= 50		N= 100		N=150	
S	Φ	MSE	MAPE	MSE	MAPE	MSE	MAPE
4	-.8	.0639305	10.87553	.008057164	5.304302	.002680294	3.816172
	-.5	.1011437	18.68727	.00967294	9.727649	.003782599	7.15672
	-.2	.1200144	45.48001	.0087438	23.82895	.00378989	17.91413
	.2	.1086968	45.1553	.009285366	24.08348	.003856349	18.06194
	.5	.07985026	18.31206	.01064271	9.818537	.00389431	7.326751
	.8	.0442770	10.21771	.007926291	5.298748	.002689877	3.860965
12	-.8	.2781982	19.84512	.02194157	6.898858	.00472844	4.412067
	-.5	.266373	31.58603	.02409929	11.9607	.007185025	8.203451
	-.2	.246023	74.13686	.0230881	29.90346	.007528732	20.33155
	.2	.2514452	75.9263	.00246051	30.42912	.00763696	20.5597
	.5	.2665016	32.17693	.2232959	12.27484	.00766359	8.413611
	.8	.2553695	19.92234	.01968032	7.000355	.00541834	4.585733

Table (7)
Log – normal Distribution $a_t \sim Log(0, 1)$

N		N= 50		N= 100		N=150	
S	**Φ**	**MSE**	**MAPE**	**MSE**	**MAPE**	**MSE**	**MAPE**
4	**-.8**	.7851071	29.27214	.04313758	22.31878	.03110797	19.39052
	-.5	.1516301	70.22134	.1136891	62.40051	.09609478	58.23092
	-.2	.1937316	206.3488	.1664056	194.7236	.1510254	187.1979
	.2	.176794	202.5126	.1670097	199.4078	.1599685	196.1485
	.5	.1128335	65.53671	.10951	65.28663	.1073455	64.85445
	.8	.03693864	23.54215	.03079922	21.80054	.02954826	21.40699
12	**-.8**	.1406624	40.2885	.0571837	26.11108	.0377932	21.49268
	-.5	.1914756	78.30042	.1225685	64.80008	.1002785	59.44038
	-.2	.2182574	215.3368	.1707164	196.6539	.1520362	187.5439
	.2	.1975366	208.8562	.1700003	200.2254	.1602461	195.8848
	.5	.1483745	72.62226	.1178736	67.30931	.1111419	65.0864
	.8	.09431124	35.96233	.05042303	27.60299	.03999342	24.7907

Table (8)
Logistic Distribution a_t ~ Lap (1,2)

N		N= 50		N= 100		N=150	
S	**Φ**	**MSE**	**MAPE**	**MSE**	**MAPE**	**MSE**	**MAPE**
4	**-.8**	.01741365	12.36673	.00707847	8.003753	.004433057	6.365224
	-.5	.02370643	23.93953	.1196252	17.22473	.008266451	14.37947
	-.2	.0261524	64.00808	.01485558	48.86301	.010190857	41.97515
	.2	.02416354	63.09167	.10536158	50.66165	.01221362	45.42171
	.5	.01857255	22.44785	.1254992	18.73359	.01084767	17.66392
	.8	.00959534	9.962268	.005951921	8.3914	.005750592	8.469648
12	**-.8**	.02757528	15.8138	.008967982	9.176959	.00488606	6.756415
	-.5	.03157759	27.52099	.01378866	18.49896	.008729639	14.79766
	-.2	.03214239	70.45515	.01624662	50.88001	.01132663	42.85431
	.2	.02850119	67.40163	.01591302	51.27511	.01241634	45.94474
	.5	.02338349	24.49599	.0127595	18.70216	.010847	17.63457
	.8	.01738493	13.08476	.000705234	8.790076	.006058303	8.52489

Table (9)

Laplace Distribution a_t ~ Lap (1,2)

N		N= 50		N= 100		N=150	
S	Φ	MSE	MAPE	MSE	MAPE	MSE	MAPE
4	-.8	.1207038	38.82751	.08665049	34.399	.08099004	33.93496
	-.5	.1065493	57.82305	.08568997	54.19956	.08199526	54.31106
	-.2	.07286004	117.7776	.06243431	114.6918	.06037554	116.0691
	.2	.03664706	80.77973	.03299808	81.215	.03208071	83.09741
	.5	.01949017	23.45166	.01704159	23.0485	.01657785	23.51059
	.8	.00722994	8.49973	.00492543	7.702899	.004940058	8.095187
12	-.8	.1638244	44.37848	.1048882	37.5537	.08749618	35.17323
	-.5	.0098785	60.30869	.09249626	56.02486	.08345146	54.53029
	-.2	.07926874	120.0353	.0659539	117.1397	.06018423	115.2187
	.2	.04168786	82.22821	.03421386	81.48963	.03127756	81.31745
	.5	.02704439	25.8107	.01742033	22.65388	.0158798	22.57115
	.8	.024636	15.08072	.00643495	8.200066	.004721297	7.413226

Table (10)

Weibull Distribution a_t ~ Wei (1,2)

N		N= 50		N= 100		N=150	
S	Φ	MSE	MAPE	MSE	MAPE	MSE	MAPE
4	-.8	.1008502	34.62887	.0361642	28.52714	.05384916	26.84512
	-.5	.1942608	82.8933	.1674922	78.8819	.1603867	78.1943
	-.2	.2463553	240.7645	.2383504	240.1232	.2370896	240.7418
	.2	.2174721	230.121	.2234844	234.9887	.2261201	236.8694
	.5	.1329158	72.35992	.1361669	73.16188	.1376127	74.08466
	.8	.04099346	25.1151	.03481232	23.28804	.03359199	22.89611
12	-.8	.168281	46.76237	.08524452	33.6663	.3641928	29.45665
	-.5	.2337902	90.97487	.1827786	82.40308	.1670937	79.70251
	-.2	.2656746	247.8949	.24540542	243.1865	.2398287	242.03
	.2	.2334541	235.4719	.2276442	236.6951	.228387	237.8922
	.5	.700171	80.48749	.1480952	76. 59925	.144559	75.87184
	.8	.1046598	39.30177	.05936206	30.29471	.0471831	27.0957

Table (11)

Pareto Distribution $a_t \sim Par(1,2)$

N		N= 50		N= 100		N=150	
S	**Φ**	**MSE**	**MAPE**	**MSE**	**MAPE**	**MSE**	**MAPE**
4	**-.8**	.1694456	17.70755	.03307659	8.576957	.009066504	6.027019
	-.5	.1310291	36.01339	.01495098	20.46413	.01393503	15.03067
	-.2	.1317036	103.0954	.41536645	63.25665	.01911737	47.7422
	.2	.1837355	110.0585	.04834323	73.26382	.02377504	57.3454
	.5	.1899569	40.46947	.05118516	28.46978	.2018319	23.0119
	.8	.138047	17.46189	.02528647	12.3017	.01082782	10.44616
12	**-.8**	1.294819	35.72892	.1095018	12.21755	.03209792	7.71781
	-.5	1.412704	62.50469	.05816617	24.33264	.04141450	17.49515
	-.2	1.412704	164.0699	.06755581	72.30277	.04932924	53.94121
	.2	1.53024	174.0145	.07529148	82.08772	.05261024	63.89881
	.5	1.486554	66.95798	.074301	32.06267	.04710209	25.53136
	.8	1.16304	36.25483	.1017756	16.21147	.037847262	12.47528

5- Conclusions:

By experimentally simulation, the following conclusions are obtained :

a- MSE and MAPE values increase when seasonal period is increased regardless the parameter Φ initial values are positive or negative.

b- MAPE and MSE values decrease when parameter Φ initial values are approaching ±1, while MAPE and MSE increase when parameter Φ initial values are approaching zero.

c- MAPE and MSE values decrease when sample size is increased with positive initial values of parameter Φ. MAPE and MSE values increase when sample size is increased with negative initial values of parameter Φ.

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المستخلص

يهدف البحث الى تتبع سلوك معلمة نموذج الأنحدار الذاتي الموسمي غير الطبيعي من الدرجة الأولى من خلال دراسة محاكاة . ويتم ذلك بأفتراض أن لأخطاء العشوائية للنموذج تتبع توزيعا مستمرا غير طبيعيا وعن طريق أستخدام أداتي مقارنة هما متوسط مربعات الخطأ MSE للمعلمةΦ ومتوسط الخطأ النسبي المطلق MAPE للمعلمة Φ ولحالات مختلفة (الفترة الموسمية ، حجم العينة ، القيمة الأولية للمعلمة Φ وأشارات القيم الأولية لهذه المعلمة) .

ان أهم الاستنتاجات هي ان قيم MSE و MAPE للمعلمة Φ تتزايد عندما تتزايد الفترة الموسمية وحجم العينة مع قيمة سالبة أولية للمعلمة Φ وللقيم الأولية للمعلمة Φ التي تقترب من الصفر . وأيضا قيم MSE و MAPE للمعلمة Φ تتناقص عندما تتناقص الفترة الموسمية (بغض النظر كون قيمة المعلمة Φ الأولية سالبة أو موجبة) ان القيمة الأولية للمعلمة Φ (± 1) وحجم العينة يزداد مع قيمة أولية للمعلمة Φ .